

C.U.SHAH UNIVERSITY

WADHWAN CITY

University (Winter) Examination -2013

Course Name :B.Tech Sem-I

Subject Name: - Engineering Mathematics-I

Marks :70

Duration :- 3:00 Hours

Date : 16/12/2013

Instructions:-

- (1) Attempt all Questions of both sections in same answer book / Supplementary.
 (2) Use of Programmable calculator & any other electronic instrument is prohibited.
 (3) Instructions written on main answer Book are strictly to be obeyed.
 (4) Draw neat diagrams & figures (If necessary) at right places.
 (5) Assume suitable & Perfect data if needed.

SECTION I

- Q-1 (a) Determine the region in z-plane represented by $\text{Re}(z) > 3$ 01
 (b) Find polar form of complex number $z = -\sqrt{3} + i$ 02
 (c) Evaluate $\lim_{x \rightarrow \pi/2} \frac{\log \sin x}{(\pi - 2x)^2}$ 02
 (d) Define absolute and conditional convergence of series. 02
- Q-2 (a) (i) Use definition of differentiation to find $f'(x)$ if $f(x) = \frac{x}{x-1}$ 02
 (ii) Find the complex number z if $\arg(z+1) = \frac{\pi}{6}$ and $\arg(z-1) = \frac{2\pi}{3}$ 03
 (b) (i) Test the convergence of series $\sum_{n=1}^{\infty} \frac{n^3+2}{2^n+2}$ 02
 (ii) Prove that $\left(\frac{1+\sin \theta+i \cos \theta}{1+\sin \theta-i \cos \theta}\right)^n = \cos n\left(\frac{\pi}{2}-\theta\right) + i \sin n\left(\frac{\pi}{2}-\theta\right)$ 03
 (c) Prove that n^{th} root of unity are in geometric progression. Also, show that their sum is zero. 04

OR

- Q-2 (a) (i) Solve the equation $z^2 - (5+i)z + 8+i = 0$ 02
 (ii) Is $\text{Arg}(z_1 z_2) = \text{Arg}(z_1) + \text{Arg}(z_2)$? Justify. 03
 (b) (i) Test the convergence of series $\left(\frac{1}{2}\right)^{1^2} + \left(\frac{2}{3}\right)^{2^2} + \left(\frac{3}{4}\right)^{3^2} + \dots$ 02
 (ii) Expand $\sin^5 \theta \cos^3 \theta$ in series of sine multiple of θ 03
 (c) Evaluate the following limits: 04
 (i) $\lim_{x \rightarrow 1} (1-x^2)^{\frac{1}{\log(1-x)}}$ (ii) $\lim_{x \rightarrow 0} \left(\frac{1}{x^2} - \cot^2 x\right)$
- Q-3 (a) (i) Evaluate $(1+i\sqrt{3})^8 + (1-i\sqrt{3})^8$ 03
 (ii) Trace the curve $y^2(2a-x) = x^3$ 04
 (b) (i) Find constants a, b, c so that $\lim_{x \rightarrow 0} \frac{x(a+b \cos x)-c \sin x}{x^5} = 1$ 03
 (ii) Test the convergence of series $\frac{x}{2.5} - \frac{x^2}{2^2.10} + \frac{x^3}{2^3.15} - \frac{x^4}{2^4.20} + \dots$ 04



OR

- Q-3 (a) (i) Find the asymptotes of curve $x^2y^2 - x^2y - 5xy^2 + x + y + 5 = 0$ 03
(ii) Test the convergence of following series: 04
(1) $\sum [\sqrt{n^2 + 1} - 1]$ (2) $\frac{1}{1 \cdot 2} - \frac{1}{3 \cdot 4} + \frac{1}{5 \cdot 6} - \frac{1}{7 \cdot 8} + \dots$
- (b) (i) Find and plot all roots of $\sqrt[3]{8i}$ 03
(ii) Find the radius of convergence and interval of convergence of series 04
 $-\frac{(x+2)}{1 \cdot 2} + \frac{(x+2)^2}{2 \cdot 2^2} - \frac{(x+2)^2}{3 \cdot 2^3} + \frac{(x+2)^3}{4 \cdot 2^4} - \dots$

SECTION II

- Q-4 (a) State Euler's Theorem on Homogeneous Function 01
(b) Find the Maclaurin's series of $\log(1-x)$ 02
(c) Find $\frac{\partial(u,v)}{\partial(x,y)}$ for $u = x^2 - y^2, v = 2xy$ 02
(d) If $u = e^{xyz}$, find $\frac{\partial^3 u}{\partial x \partial y \partial z}$ 02
- Q-5 (a) (i) Prove that $\sqrt{1 + \sin x} = 1 + \frac{x}{2} - \frac{x^2}{8} + \frac{x^3}{48} + \dots$ 02
(ii) If $u = f\left(\frac{x}{y}, \frac{y}{z}, \frac{z}{x}\right)$, prove that $x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} + z \frac{\partial u}{\partial z} = 0$ 03
- (b) (i) Find the linearization of $f(x, y, z) = x^2 - xy + 3 \sin z$ at the point $(2, 1, 0)$. 02
(ii) Use partial derivatives to find $\frac{du}{dx}$ if $u = \tan^{-1}\left(\frac{x}{y}\right), x^2 + y^2 = a^2$ 03
- (c) If $x = \frac{\cos \theta}{u}, y = \frac{\sin \theta}{u}$, evaluate $\left(\frac{\partial x}{\partial u}\right)_\theta + \left(\frac{\partial u}{\partial x}\right)_y + \left(\frac{\partial y}{\partial u}\right)_\theta + \left(\frac{\partial u}{\partial y}\right)_x$ 04

OR

- Q-5 (a) (i) Find Taylor's series expansion of $f(x) = x^3 - 2x + 4, a = 2$ 02
(ii) If $w = xy + yz + xz, x = u + v, y = u - v, z = uv$, find $\frac{\partial w}{\partial u}$ & $\frac{\partial w}{\partial v}$ at $(u, v) = \left(\frac{1}{2}, 1\right)$ 03
- (b) (i) Find the equation of tangent plane and normal line to the surface 02
 $x^2 + 2y^2 + 3z^2 = 12$ at the point $(1, 2, -1)$
(ii) Discuss the continuity of $f(x, y) = \begin{cases} \frac{x^3 - y^3}{x^2 + y^2}, & (x, y) \neq (0, 0) \\ 0, & (x, y) = (0, 0) \end{cases}$ at $(0, 0)$. 03
- (c) If $u = \sin^{-1}\left(\frac{\frac{1}{x^4 + y^4}}{\frac{1}{x^5 + y^5}}\right)$, find $x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y}$ and $x^2 \frac{\partial^2 u}{\partial x^2} + 2xy \frac{\partial^2 u}{\partial x \partial y} + y^2 \frac{\partial^2 u}{\partial y^2}$ 04
- Q-6 (a) (i) If $V = r^m$, where $r^2 = x^2 + y^2 + z^2$ then prove that 03
 $V_{xx} + V_{yy} + V_{zz} = m(m+1)r^{m-1}$
(ii) Expand $\tan^{-1}\left(\frac{y}{x}\right)$ in powers of $(x-1)$ and $(y-1)$ using Taylor's expansion. 04



- (b) (i) Find percentage error in area of an ellipse when errors of 2% and 3% are made in measuring its major and minor axes respectively. 03
- (ii) A soldier placed at point (3,4) wants to shoot the fighter plane of an enemy which is flying along the curve $y = x^2 + 4$ when it is nearest to him. Find such distance by using Lagrange's multipliers method. 04

OR

- Q-6 (a) (i) Expand $\sin^{-1} x$ up-to the first four terms by Maclaurin's series. 03
- (ii) Show that $JJ' = 1$ for $x = e^v \sec u$, $y = e^v \tan u$ 04
- (b) (i) Find the extreme values of function $x^2y - xy^2 + 4xy - 4x^2 - 4y^2$ 03
- (ii) If $f(xy^2, z - 2x) = 0$, show that $2x \frac{\partial z}{\partial x} - y \frac{\partial z}{\partial y} = 4x$ 04

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